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A FINITE ELEMENT METHOD FOR ENVIRONMENTAL AND POLLUTANT TRANSPORT PREDICTION

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Abstract

In the last decades, significant researches were dedicated to model the water-quality trends, soil erosion and material losses, contaminant transport, and their ecological impacts. The present paper investigates theoretically and presents a numerical model with finite elements of the pollutant transport and diffusion in 2-D environmental flow. It is a general model based on the physical groundwater flow equations presented into a new approach, in quantity structure. The mathematical model is based on mass conservation, including the main flow of the fluid and existing pollutants. The fluid mass conservation includes the changes in the groundwater volume flow in a selected time interval and the appeared modifications in the aquifer characteristics due to the pollutant presence are used in the numerical model. Some supplementary considerations concerning mathematical modeling are briefly presented. This model can be used to simulate the groundwater behavior in different situations, with a concentrated source of the pollutant, or with 2 sources of pollution. In the present paper are simulated only the aquifers with mainly horizontal groundwater flow and with a relatively known aquifer's structure. The model is tested for a landfill near the Dimbovita River, to predict the groundwater contamination and the time variation of concentration. Finally, some conclusions and references are presented.

Keywords*: Environmental engineering, Mathematical model, Numerical modeling, Quality of water, Water pollution*

Introduction

The pollution of the aquifers, pollutants transport and dispersion due to improper waste management or due to different accidents in mines has become an actual problem for the Romanian authorities. The accelerated process of urbanization and industrialization, improvement of the standard living conditions, population growth, and fast economical development are sources of a large number of waste materials, needed to be deposited (Kontos et al. 2005), (Rojanschi et al. 2002). Mines that have not been rehabilitated for many years, the tanks of deposits, the sorting and washing stations are another possible source of pollution; it should not forget the environmental accident, five years ago, when they were discharged harmful substances in river Crisul Repede. Nowadays, waste management strategies should consider the public health and environmental protection. In Romania, generally, the main waste deposits are near to river courses, for example, Olt, Crisul Repede, Crisul Negru, Dambovita, Prahova, etc. In these conditions, if there are not deposited properly, for deposits older than 12 years, they can pollute the groundwater aquifers. In rainy years, as it is 2019, due to the water infiltration may appear a rapid transport of pollutants in the aquifer, closely related to the type of soil, (Romanian law 211 2011), (Normativul tehnic 150 2003).

At European Union level, the waste management is based on some key principles, such as prevention of waste generation, recycling and reuse, improving and monitoring of waste disposal methods. The final waste disposal in landfills must be considered only after having applied preventive measures regarding waste generation and recovered useful materials and energetic power. The studies on the suitability of the waste landfill emplacement are particularly important, as they provide landfill design and construction models considering environmental protection. Various international specialized studies were conducted to identify suitable areas for waste landfill location, several using GIS techniques. In Romania, several studies on waste landfill sitting requirements were completed, but waste management is still being inadequately implemented.

According to the latest Eurostat report, which includes data considering waste management, 96% of the collected municipal waste in Romania is disposed of in dumps and landfills, while only 4% is recycled. Romania is ranked among the last places in the EU regarding municipal waste recycling. Therefore, the management of the waste disposal must consider the restrictive environment variables, which can be numerically modeled, and efficiently analyzed.

Materials and Methods

Once it has appeared pollutant in soil, due to its permeability and of water infiltration, dispersion occurs. The phenomenon can be described as in Fig. 1-a, and in Fig. 1-b the pollutant transport through soil porosity up to the aquifer (Kjeldsen et al. 2002).

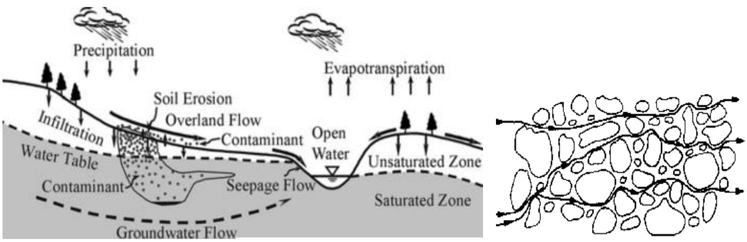

Figure 1. Pollutant transport

The field is considered an unsaturated porous medium. In these conditions, the mass transfer equation in an elementary volume can be written:

$$div\left[\theta(x,y,z,t)\overline{\overline{D}}(x,y,z,t)\rho(x,y,z,t)\left(grad\frac{C(x,y,z,t)}{\rho(x,y,z,t)}\right)\right]-$$
$$-div\left[\vec{U}(x,y,z,t)C(x,y,z,t)\right]=\frac{\partial}{\partial t}\left[\theta(x,y,z,t)C(x,y,z,t)+\rho_b S\right]+\mu_w\theta(x,y,z,t)C+$$
$$+\mu_s\rho_b S-\mu_w\theta(x,y,z,t)-\mu_s \qquad (1)$$

Where: C(x,y,z,t) [kg/m^3]- the volumetric concentration of pollutant, D(x,y,z,t) [m^2/s] – the dispersion coefficient of pollutant in the aquifer water, S- punctual source of pollution, ρ_b(x,y,z,t) [kg/m^3]-density of the mixed composition (soil and water), U(x,y,z,t) [m/s] – the components of velocity, θ - the soil humidity, μ_w[s^{-1}]- the decreasing rate of pollutant's concentration in the liquid phase of the porous soil, and μ_s [s^{-1}]- the decreasing rate of pollutant's concentration in the solid phase of the porous soil [6]. After some calculations and noting:

$$\mu=\mu_w+\frac{\mu_w\rho_b K_d}{\theta}\,; \qquad \mu_1=\mu_s+\frac{\mu_s\rho_b}{\theta} \qquad (2)$$

Where K_d is a coefficient depending on the pollutant type. Finally, equation (1) became:

$$\frac{\partial}{\partial t}(C\theta R)=div\left[\theta(x,y,z,t)\overline{\overline{D}}(x,y,z,t)\rho(x,y,z,t)\left(grad\frac{C(x,y,z,t)}{\rho(x,y,z,t)}\right)\right]-$$
$$-div\left[\vec{U}(x,y,z,t)C(x,y,z,t)\right]-\theta C_\mu+\theta\mu 1 \qquad (3)$$

A practical solution to pollutant transport in an aquifer is a complex problem and can be solved in two ways:

1. Density and viscosity of the mixture are considered constant in time and space.

The hydrodynamic equations are independent of the dispersion equation (3) because density and viscosity do not depend on C. To be useful such modeling it must be updated at a time step no more than two days, based on local measurements (Schiopu, & Gavrilescu 2010). In this case, the pattern of dispersion is reduced at a system of partial differential equations and the working steps will be the following:

- are solved the hydrodynamic equations, the Darcy equation, and the continuity equation, taking into account the boundary conditions and initial conditions. It is obtained the velocity distribution in space and time, for different time intervals
- the coefficients of the dispersion equation, are expressed depending on the velocity at each time step
- the simplified equation of dispersion may be solved

There is obtained the distribution of concentration, space and time, for all time steps.

2. The general case, $\rho=\rho(C)$, $\eta=\eta(C)$. The velocity and the dispersion coefficients depend on concentration. It can not be obtained simultaneously the solutions for all equations. To solve the dispersion problem are considered the next steps:

a) Are made some simplifying assumptions:

- The distribution of concentration is known at time t; then corresponding η and ρ will be estimated
- It is assumed that for a time interval dt, small, the C, R, and h are constants

b) It is calculated the velocity distribution at time t + dt

c) It is estimated the concentration at time t + dt

d) All parameters are calculated for time t + dt.

With the forward-backward method, all necessary parameters will be determined, assuming that the dispersion tensor must be decomposed in three directions x, y, z to make the integration.

To solve the problem of the pollutant dispersion in a porous medium involves the simultaneous integration of the equations describing the water flow into the soil pores and the equation of pollutant dispersion. The coefficients of the dispersion are linear functions, variables in time and space, depending on the flow regime. The permeability coefficients are nonlinear functions depending on the soil characteristics, and the density and the dynamic viscosity depend on the solution concentration. It is considered the dispersion problem in the soil near a river, with higher soil moisture.

The problem is solved in the next steps:

- Dispersion problem is solved as previously stated in paragraph two
- Are collected some samples of the soil for proper calibration of the model: the soil permeability, flow discharge in the river, rainfall amount, type and concentration of pollutant, etc.
- It is predicted the phenomenon.

Further will be analyzed a 2D-dispersion problem, near a river. The x, y directions are considered perpendicular, then the dispersion equation may be written:

$$\frac{\partial C}{\partial t} = D_{xx}\frac{\partial^2 C}{\partial x^2} + 2D_{xy}\frac{\partial^2 C}{\partial x^2} + D_{yy}\frac{\partial^2 C}{\partial y^2} - v_x\frac{\partial D}{\partial x} - v_y\frac{\partial D}{\partial y} - \lambda C - \sum Q_i\delta_i \quad (4)$$

Where:

$$D_{xx} = D_l\frac{v_x^2}{v^2} + D_S\frac{v_y^2}{v^2}; \qquad D_{yy} = D_S\frac{v_x^2}{v^2} + D_l\frac{v_y^2}{v^2};$$

$$D_{xy} = (D_l - D_S)D_l\frac{v_x v_y}{v^2} \quad (5)$$

After a few calculations, the equation (4) may be transformed and integrated for solving the specific problem. A quantity of water may be directly added or removed from the aquifer in top or bottom, due to the processes of infiltration, rainfall or vertical leakage. The mathematical model assumes that the vertical water added in the aquifer immediately turns and merges with the horizontal flow of the groundwater (Oroian, & Odagiu 2012). The equation of mass conservation is:

$$D\frac{\partial h}{\partial t} = \frac{\partial}{\Im x}\left(\lambda\frac{\partial h}{\partial x}\right) + \frac{\partial}{\Im y}\left(\lambda\frac{\partial h}{\partial y}\right) + Q + Q_l \quad (6)$$

Where: h (x ,y, t) – the hydraulic head in the aquifer, λ- the transmissivity for a confined aquifer, Q(x,y) - the strength of a source function, representing the fluid volume per time, added per horizontal area of the aquifer, Q_l(x,y) – the liquid pollution leakage into the aquifer, equal with:

$$Q_l = -C(h - h_i) - \frac{k}{l}(h_i - h_A) \quad (7)$$

Where h_i-(x,y)–initial hydraulic head in the aquifer, h_A(x,y)–hydraulic head after pollutant admission, k(x,y) – hydraulic conductivity, l(x,y) – thickness layer.

$$h(x,y,t) = \sum_{j=1}^{N} H_j(t)\phi_j(x,y) \tag{8}$$

Where H_j-the hydraulic head in node j and $\phi_j(x,y,t)$- the node j characteristics, the basic functions. In the Galerkin method, the residue is forced to be orthogonal to all basic functions and thus, the average becomes zero over the entire mesh of finite elements. This implies N orthogonally requirements in the interest area. Terms of space derivative may be integrated:

$$\iint_A \left(D\frac{\partial h}{\partial t} - Q - Q_l \right)\phi_i(x,y)dxdy + \iint_A \lambda\left(\frac{\partial h}{\partial x}\frac{\partial \varphi_i}{\partial x} + \frac{\partial h}{\partial y}\frac{\partial \varphi_i}{\partial y} \right)dxdy -$$
$$- \int_B \lambda\left(\frac{\partial h}{\partial x} cos\, x + \frac{\partial h}{\partial y} cos\, y \right)\phi_i(x,y)dx = 0 \tag{9}$$

Where cos(x), cos(y) are the cosine directions between the outward normal to the output boundary B, A- input boundary area.

Integration of the two-dimensional dispersion equation is performed by alternating directions. It is considered the punctual pollutant, from a source of known concentration. Integrating equation (5) must be made imposing one unique solution

- initial conditions, input A boundary area $C(x, y, t) = C_i(x,y)$, t=0
- boundary conditions
    - vertical input cross section, $C(x=0, y, t) = C_f(o,y,t)$- known
    - vertical output cross section, $C(x=Lx,y,t) = C_f(Lx,y,t)$- known
    - horizontal upper side aquifer, $C(x,y=Ly/2,t) = C_f(x,Ly/2,t)$- known
    - horizontal bottom aquifer, $C(x,y= - Ly/2,t) = C_f(x,-Ly/2,t)$- known

The meshing system, at time t + dt/2

$$C^{t+1}(i,j-1)(p+s) + C^{t+1}(i,j)(-2p-2) + C^{t+1}(i,j+1)(p-s) =$$
$$= C^t(i-1,j)(-c-q) + C^t(i,j)(2c-2) + C^t(i+1,j)(c-q) - dtS^{t+1}(i,j) \tag{10}$$

Where

$$p = \frac{D_l dt}{dx^2}; \qquad s = \frac{v_x dt}{2dx}; \qquad c = \frac{D_{tl} dt}{dy^2}; \qquad q = \frac{v_y dt}{2dy} \tag{11}$$

And for time moment t + dt

$$C^{t+2}(i-1,j)(c+q) + C^{t+2}(i,j)(-2c-2) + C^{t+2}(i+1,j)(-c+q) =$$
$$= C^{t+1}(i,j+1)(-p-s) + C^{t+1}(i,j)(2p-2) + C^{+1t}(i,j+1)(-p+s) - dtS^{t+2}(i,j) \tag{12}$$

Results and Discussion

In Romania, there are currently 68 landfills that are not complying with the European regulations concerning the ground storage, their sealing, places where they have been allocated and finally the consequences on the environment. Of these 32 are in the vicinity of the rivers. There are also industrial basins of washing associated with mines that have not been rehabilitated for more than 20 years. Five years ago was recorded an ecological accident, when contaminated water has been infiltrated into the ground, with major consequences on agricultural production.

In present paper is analyzed the variation in time and space of the pollutant concentration derived from a landfill that continually infiltrates into an aquifer, for long time-interval. The analyzed landfill is located near the river Dambovita, Fig. 2; the market is the modeled area.

Figure 2. A landfill near Dambovita River

The sizes of the aquifer are L_x =200 m, L_y =1000 m, the flow rate in aquifer is 0.1 m/day, longitudinal dispersivity α_L= 2 m, and transverse α_T = 20m, concentration at source C_1=160 mg/l, flow velocities in aquifer v_x=0,12 m/day, v_y=0,038/day, concentration at the initial moment C(x,y,0) = 0,00012 mg/l, at the domain frontiers at initial moment C_F(x,y,0) = 0, and the injection flow rate is 0.002 m/day.
In Fig. 3, Fig.4 are presented in detail only the areas where the infiltration occurs, during the time. In Fig. 3 is shown the variation in concentration at the interval of three months for the first year and in Fig. 4 the predicting model for ten years. Curves represent equal concentration for different time intervals.

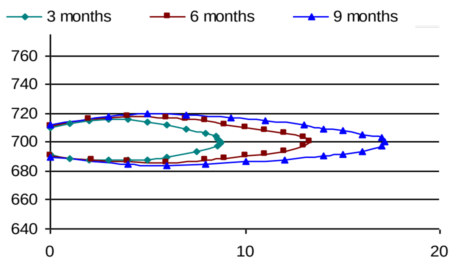

Figure 3. Dispersion of pollutant material for the first nine months

During the numerical modeling, it was observed that for t=1 day there are no significant changes in the dispersion of the pollutant. The first significant results were observed after 5-6 days. It has therefore selected a minimum gap of 7 days. The numerical solution is stable and convergence. The change does not alter the time step for the obtained solution and was chosen dx= 5 m and dy=20 m. In the first year, the flow is more pronounced in the x-axis direction.

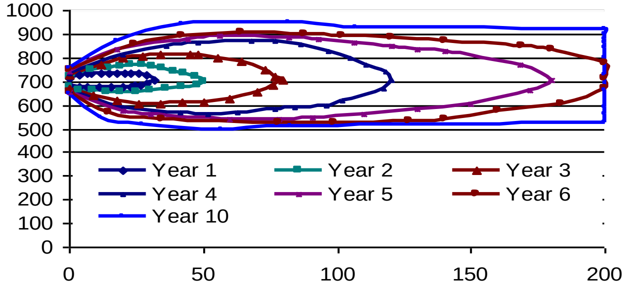

Figure 4. Dispersion of the pollutant material for ten years

Starting the second year of pollution due to the soil permeability the shape of the pollutant area tends to increase its width and the shift direction through x direction diminishes. From the third year, there are only predictive results. Starting the sixth year, the pollutant reaches the watercourse. If it is extended the analysis to a larger area, after about 30 years the problem becomes stationary. In Fig. 5-a is shown the concentration variation in the direction of axis X for ten years, and Fig. 5-b has compared the numerical results with experimental results for the first three years; since the third year, there are only numerical results.

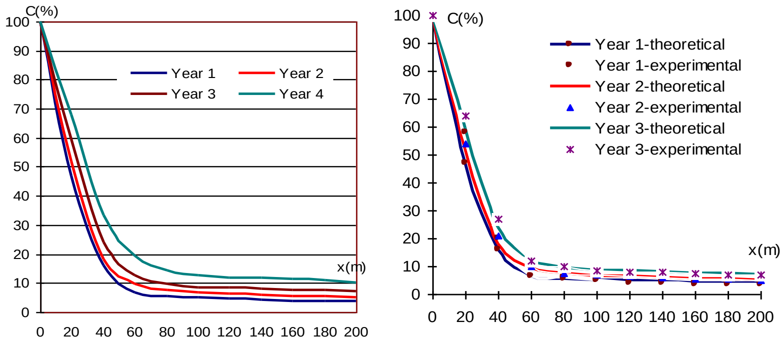

Figure 5. Concentration variation in direction x

Step's change for the time, in a reasonable way, does not alter the curves allure. Any modification for steps on x-direction does not influence on curves distribution. The concentrations on the domain border, considered as the limit conditions does not fundamentally alter over time.

Conclusions

Using the finite element method was determined the velocity distribution of the pollutant transport and the variation of concentration on x-direction.

If there is horizontal bedrock in the domain's bottom, the streamlines will be modified and will appear a different distribution of the concentration dispersion.

Utilization of the proposed model assumed some essential steps: determination of the grid mesh for finite element method, estimation of the aquifer concentration and its position, knowledge of the hydrologic regime, boundary conditions, etc.

They have traced the equal concentration curves for different intervals of time and have been compared the numerical results with the experimental measurements. After 20 years it is estimated that the pollutant front moves about 450 m. By comparing the analytical and numerical results it was observed that for a distance greater than 400 m from the source, the differences are negligible.

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