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CPM AND PERT TECHNIQUES FOR SMALL-SCALE R&D PROJECTS

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Abstract

Contemporary Project Management has conceived tools based on mathematical models for planning, scheduling and controlling the projects, the costs and resources. Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) are two network-based methods which were independently developed to assist the project managers in order to schedule complex real-life projects. A scheduling of a small-scale R&D project using these optimization time-oriented methods was accomplished. In many models of project network some activities are closely related to each other such as procurement activities of basic resources and research activities. If this precedence relationship is on the critical path, that means without no event slack or float time for activities, then some procurements delay may cause lag in the project's completion time. Thus a good estimation for procurement and research activities duration is needed for a Just-in-Time project.

Keywords: *CPM, network scheduling, PERT, procurement, project management*

Introduction

Planning is the most important stage of project management. In this phase are established objectives of the project, well-defined tasks or activities, resources and are estimated the costs and durations. Scheduling refers to the allocation of resources in terms of space, time and effort. Techniques usually used for planning and scheduling are network-based methods. The network diagram is a graphical description of logical relationships among project activities. The project plan is a graph where the sequence of activities is drawn as chains of nodes and arrows. Many various of planning tools have been developed such as CPM, Metra Potential Method, PERT, Precedence Diagram Method, Critical Chain Project Management etc. In the following sections data processing have been performed by using CPM and PERT. In the late 50s Walker from DuPont Company and Kelley of Remington Rand established mathematical foundations of the Critical Path Method (Kelley & Walker 1959). They solved the problem of cost optimisation by parametric linear programming. It is a problem of optimum path in a graph to maximize a value. In the same time, the USA Navy has succeeded to find new technique to carry out their military projects. PERT was developed in order to support the U.S. Navy's Polaris missile program (Malcolm et al. 1959). The network approach of the project planning and controlling continues to be widely used today.

Materials and Methods

Methods of project planning

Project Schedule Management covers the processes required to manage the timely completion of the project. According to PMBOK methodology the processes are: Plan schedule management, Define activities, Sequence activities, Estimate activity durations, Develop schedule and Control schedule (PMBOK® Guide 2017).

Critical Path Method (CPM)

Critical Path Method is a step-by-step schedule technique which estimates the minimum project duration on the logical network paths. Mathematically speaking, a form of the Ford-Fulkerson network flow algorithm is used to solve this scheduling problem. The simplified Ford algorithm applied to directed acyclic graph with one start and one finish node is an algorithm for the shortest-path problem. Three types of information are needed to describe any project: activities, relationships and durations (Hillier & Lieberman 2015). Activities of the project graph plan are represented by arrows and events are depicted by nodes as seen in Figure 1. Events mark out the logic between activities.

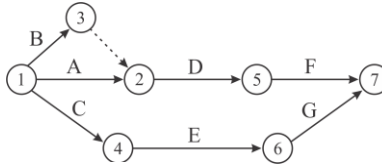


Figure 1. Typical project network diagram

CPM technique calculates the early start, early finish, late start, and late finish dates for all activities by performing a forward and backward pass analysis through the project network (Figure 2). The necessary elements for CPM time computation are the sequence of activities, dependencies between them and estimated duration of each activity.



Figure 2. Activity earliest and latest times

Work Breakdown Structure (WBS) could be used to build the activities found in the project network (Larson & Gray 2017). A single-point estimating for activity duration is accomplished. The project network with events and activities is designed. There are two types of diagrams: Activity on Arrow (AoA) and Activity on Node (AoN). The CPM analysis for an AoA network approach follows two steps:

Forward pass

The earliest event and activity times within the network are computed

$$EET(j) = \max [(EET(i) + d(i,j))] \quad (1)$$

$$EST(i,j) = EET(i) \quad (2)$$

$$EFT(i,j) = EST(i,j) + d(i,j) \quad (3)$$

Backward pass

The latest event and activity times in the network are calculated

$$LET(i) = \min [LET(j) - d(i,j)] \tag{4}$$

$$LFT(i,j) = LET(j) \tag{5}$$

$$LST(i,j) = LFT(i,j) - d(i,j) \tag{6}$$

where: EET is earliest event time, EST is earliest starting time, EFT is earliest finishing time, LET is latest event time, LST is latest starting time, LFT is latest finishing time, d is activity duration.

Total float of activity (i,j)

$$TF(i,j) = LFT(j) - EST(i) - d(i,j) \tag{7}$$

Free float of activity (i,j)

$$FF(i,j) = EST(j) - EST(i) - d(i,j) \tag{8}$$

The float time of activity is the maximum time interval that the activity can be delayed without slowing the project completion. The path whose length is equal to the minimum duration of the project is called critical path. Activity on the critical path and any activity that does not have a float time is called critical activity.

Program Evaluation and Review Technique (PERT)

PERT is a technique appropriate for projects where the time needed to complete the activities are not known well. PERT uses a probabilistic approach while CPM is a deterministic model. In network activity, durations are defined by stochastic variables. The distribution of activity durations follows a beta distribution with parameters: min, max and mean value. PERT-beta distribution is defined by the three point estimate illustrated in Figure 3 where:

to = optimistic time (under the most favorable conditions)

tm = most likely time (most probable length of time)

tp = pessimistic time (under the worst conditions)

The mean value or expected duration *te* of activity is

$$te = (to + 4tm + tp) / 6 \tag{9}$$

the variance (square of the standard deviation σ) is

$$V = (tp - to)^2 / 36 \tag{10}$$

The Beta distribution was chosen by original PERT team for a simple reason:

"...the analysis requires some model for the distribution of activity times, the parameters of the distribution being the mode and the extremes. The distribution that first comes to the author's mind is the beta distribution" (Clark 1962).

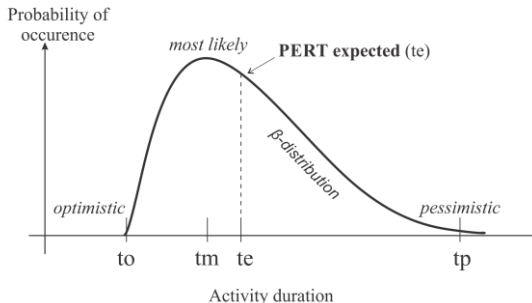


Figure 3. Density function of the PERT-beta distribution

The PERT-beta distribution has been criticized by many researchers and many distributions were suggested. The effect of different activity distributions on the project duration, such as PERT-beta, triangular or uniform distribution does not result in significant differences from a practical point of view (Hajdu & Bokor 2014). The three-point estimate is a better estimate compared to a single-point estimate. In the PRINCE2 methodology approach, estimating techniques can be top-down and bottom-up, comparative and parametric estimating, single-point or three-point estimating or Delphi technique (PRINCE2® 2017).

Results and Discussion

Obviously, in R&D projects many experimental activities cannot begin without necessary equipments, materials or some services. In the project network the procurement activities P_1, P_2, \dots, P_i will precede most of the R&D activities as seen in the diagram from Figure 4. They are closely related to each other.

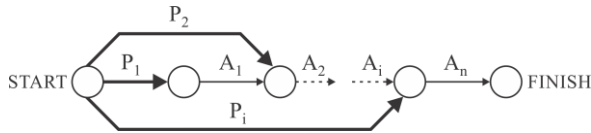


Figure 4. Precedence relationship between two activity types P and A for an ordinary one-chain project network

The research activities are provided by Work Packages (WP) from WBS or Product Breakdown Structure (PBS). The network from Figure 5 representing research work packages is condensed in a R&D subnetwork D noted WP (ij) but the precedence is still preserved.

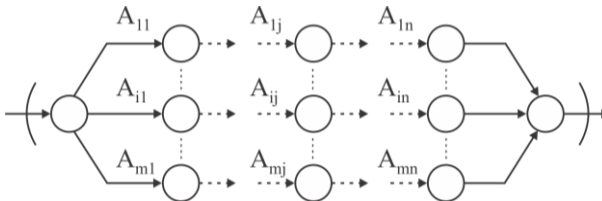


Figure 5. Research project activities WP (ij)

Numerical Case Study

We propose for study a typical small-scale R&D project with CPM and PERT without considering resource constraints.

Data processing with CPM Method

The usual activities of a sample project with descriptions, durations and precedence relationships are listed in the Table 1. The duration of activity is a single-point estimation. The type of logical relationships is Finish-to-Start (FS) without lag.

Project Network Diagram of the project's schedule is shown in Figure 6. AoA representation requires three fictitious activities called dummy activities of zero duration for not distorting the project network logic.

Table 1. List of activities with precedence relationships for sample project #

Project activity	Immediate predecessors	Estimated duration (days)	Activity description
A	-	20	Theoretical part
B	-	50	Various acquisitions
C	-	30	Basic procurements
D	A, C	30	Experimental part WP (ij)
E	D	5	Completion of experiments
F	D	20	Results dissemination
G	D	10	Final report for project closure

The data are provided by forward and backward step calculus.

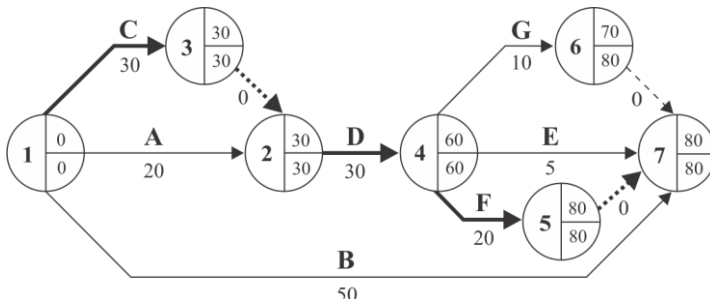


Figure 6. The AoA network diagram for sample project #

FORWARD PASS calculation

The computations of the earliest time of events and activities start from the first node 1 and advance recursively to final node 7 by using formulas (1), (2) and (3). The results can be seen in Table 2.

Table 2. Earliest Times of events and activities in Forward Pass

Event	Earliest Time of Events		Earliest Time of Activities		
	Activity finished in node	EET (j)	Activity	EST (i,j)	EFT (i,j)
1	-	0	A	0	20
2	A,C	30	B	0	50
3	C	30	C	0	30
4	D	60	D	30	60
5	F	80	E	60	65
6	F,G	70	F	60	80
7	B,E,F,G	80	G	60	70

BACKWARD PASS calculation

In Table 3 are determined latest times by starting at the end of network towards beginning of the diagram with Eq. (4), (5) and (6).

Table 3. Latest Times of events and activities in Backward Pass

Event	Latest Time of Events		Latest Time of Activities		
	Activity started from node	LET (i)	Activity	LFT (i,j)	LST (i,j)
7	-	80	A	30	10
6	-	80	B	80	30
5	-	80	C	30	0
4	E,F,G	60	D	60	30
3	D	30	E	80	75
2	D	30	F	80	60
1	A,B,C	0	G	80	70

The critical activities and float times are calculated in Table 4 by Eq. (7) and (8).

As seen in table 4 the critical events are: 1→3→2→4→5→7 and **critical activities**: C, D, F and they define the **Critical Path** with a thick line in Figure 6. Total float (TF) is the amount of time that an activity can be delayed without increasing the project completion time. Free float (FF) gives the time that an activity can be delayed without affecting any of the activity that follows.

Table 4. Critical activities and Float times

Act.	Arcs	Activity duration	Starting Time		Finishing Time		TF	FF
			EST	LST	EFT	LFT		
A	(1,2)	20	0	10	20	30	10	10
B	(1,7)	50	0	30	50	80	30	30
C	(1,3)	30	0	0	30	30	0	0
D	(2,4)	30	30	30	60	60	0	0
E	(4,7)	5	60	75	65	80	15	15
F	(4,5)	20	60	60	80	80	0	0
G	(4,6)	10	60	70	70	80	10	10

The calculated project duration is 80 days and the total floats are: 10 days for activity A, 30 for B, 15 for E and 10 for G. Generally speaking, the float indicates how much allowance each the activity has. The project was also scheduled in MS Project as seen in the Gantt chart (Figure 7).

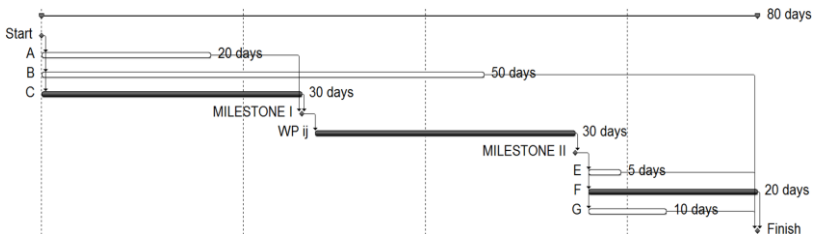


Figure 7. Tracking Gantt in MS Project software

Naturally, two milestones were proposed one before starting of experimental activities $D \cong WP (ij)$ (after basic procurements are finished) and second after research completion (for results evaluation) associated with events 2 and 4.

Data processing by PERT method

In Table 5 are determined the mean time (expected) and variance with Eq. (9) and (10). Results revealed after computations that the Critical Path is C→D→F.

Table 5. PERT computations of Mean Value and Variance

Activity	Predecessors	Optimistic time (to)	Most likely time (tm)	Pessimistic time (tp)	Mean Time (te)	Variance (V=σ ²)
A	-	15	20	30	20.83	6.25
B	-	35	45	50	44.17	6.25
C	-	20	30	40	30.00	11.11
D	A, C	20	25	35	25.83	6.25
E	D	3	6	10	6.17	1.36
F	D	15	20	25	20.00	2.78
G	D	5	8	12	8.17	1.36

The project expected duration $m = 75.83$ days is equal with sum of the expected te along critical path. Sum of the variance along the critical path is 20.14. The probability factor:

$$Z = (Ts - m) / \sigma \tag{11}$$

If the scheduled time $Ts = 80$ days, then normal distribution function finds a probability of 82.34 corresponding to $Z = 0.9285$ with Eq. (11). Hence there is 82.34 % probability of the project completion in 80 days. According to *Central Limit Theorem* even if the duration of each activity follows a β -distribution, the total duration of the project is a random variable with normal distribution.

Public Procurement Procedures

If the project is subject to public procurement legislation then the durations of project procurement activities have to be sized with the timescales of procurement procedures. For short-time R&D projects the procurements must be treated carefully if they are under this legislation. Public procurement in the European Union (EU) is regulated by a set of Directives such as Directive 2014/24/EU. The award of public contracts has to comply with principles of non-discrimination, mutual recognition, proportionality and transparency. The Public Procurement System has simplified procedures. Even so, for public contracts above a certain value the appeal procedures may extend procurement duration (Georgieva 2017). The review procedures and remedies under Directive 2014/23/EU could lead to the automatic suspension of a procurement procedure.

Modern intelligent systems in Project Management

Artificial Intelligence (AI). The environment dynamics have a great impact on projects constraints increasing their complexity becoming similar to Complex Adaptive Systems which are nonlinear dynamical systems. The project could be scheduled in Time-Constrained approach (Bodea et al. 2010) by using multi-agent methods as Swarm Intelligence Algorithms such as Genetic Algorithm or Ant Colony Optimization.

Artificial Neural Network (ANN). Artificial neural networks are complementary technologies as data mining tool used for pattern recognition, time series analysis,

prediction and clustering. The input variables may come from an enterprise's internal database such as ERP system (Relich & Muszynski 2014).

Fuzzy project management. Fuzzy logic is used to improve the classic CPM/PERT planning, with considering fuzzy activity durations and resources (Lootsma 1989), through different heuristic and metaheuristic algorithms.

Conclusions

The result of this research is scheduling of a typical small-scale project without constraints by implementation of PERT/CPM techniques. The Critical Path and critical activities were defined. The calculation refers to project duration and the total and free floats of each activity of the project.

For small-scale R&D projects, some activities are closely related to each other such as procurement activities of basic resources and research activities. This sequence of activities could be on the critical path. This fact highlights that any procurement delay may cause a lag not only in the following R&D activity but also in the completion date of the project.

If the contract above a certain value is subject to public procurement legislation, then this situation increases uncertainty in the process of estimation of the project activity duration. The Public Procurement System, regulated by a set of Directives, has few procedures that are not contested. The means of appeal, challenge and complaint, could extend the procurement durations or even suspend the contract and stop the project until proper settlement. Thus a good estimate for procurement and research activities duration is sometimes difficult for a Just-in-Time project.

Someone may think that flair and intuition are enough to plan and control smaller projects, but common things like estimation of project duration and activities floats or allocation of resources and leveling can not be easily achieved without project management scheduling techniques.

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